International Journal of Theoretical Physics, Vol. 34, No. 4, 1995

Models of Particle States

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Received July 1, 1994

Two different types of particle state models are discussed. In the first type, particles are considered to be dynamically bound systems of a small set of physical constituents. In the second type, particle states are constructed from tensor products of "symmetry constituents," i.e., states that are the basis elements of finite irreducible representations of an internal algebra. These states need not represent physical particles. We present three models of the first type. For the second type, we discuss in detail the main thrust of this paper, a new version of the quark–lepton model based on the algebra $su(4)_{\rm color} \times su(6)_{\rm flavor}$. The quark color-triplet and a lepton color-singlet are united by a single irreducible representation of $su(4)_{\rm color}$. The $su(6)_{\rm flavor}$ algebra is an extension of the original $su(3)_{\rm flavor}$. All observed ground-state hadron multiplets are in full accord with the predictions of this model. The numbers of hadron ground states it predicts are 36 spin-0 mesons, 36 spin-1 mesons, 70 spin-1/2 baryons, and 56 spin-3/2 baryons.

1. INTRODUCTION

In this paper we discuss two types of particle state models. In the first type, all particles are considered to be dynamically bound systems of a small set of basic physical constituents. In the second type, particle states are constructed from tensor products of basic spin eigenstates that belong to fundamental irreducible representations (FIR) of an internal algebra. The basic states need not represent physical particles, and may be best characterized as "symmetry constituents."

The two types of models can lead to the same set of states, as illustrated in the simple case of the H atom. The observed $so(4)$ symmetry of the levels of the H atom lead to four "H-quarks" as the elements of the 4-dimensional

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0020-7748/95/0400-0493$07.50/0

FIR of the $so(4)$ algebra. Indeed all states of the H atom can be constructed from the tensor product of these four "H-quarks." There is no potential in this model, and the states are obtained as normal modes of the H-atom Hamiltonian. On the other hand, we have the dynamical picture of the H atom consisting of proton and electron bound by electromagnetic interactions. Similar dynamical symmetry algebras, like $o(5)$, $u(6)$, . . . , have been used in nuclear and atomic physics without assuming that their FIRs are real physical constituents.

Although the two types of models can lead to the same set of states of a compound system (CS), the calculations involved are very different. In the first type one must specify the interactions of the physical constituents employing a formalism such as a gauge field theory. For strong interactions, and for a system of more than two constituents, this is usually a very difficult problem. Even for two interacting constituents the relativistic formulation is difficult.

In the second type, the basic approach is that of assuming a dynamical algebra that determines the ground states of the CS, the towers of excited states based upon them, and the transition operators between the different states. The *symmetry* constituents of the CS are the basis elements of irreducible representations (IR) of the dynamical algebra that are spin eigenstates, but have no other physical attributes, such as mass, position, momentum, or magnetic moment.

What is observed are the physical properties of the CS, and not those of the assumed constituents; and these properties can be completely described by the quantum numbers (QN) of the whole system. This is also in accord with Heisenberg's original approach to quantum mechanics, and his philosophical views in his last years for replacing the description of microscopic systems in terms of physical constituents by a global approach.

The attempt to understand the strange hadrons in the 1960s led to the $su(3)_f$ model (Gell-Mann and Ne'eman, 1964), where f stands for "flavor." This model explained well the strange hadron multiplets and some of their properties. The extension of this model in the 1970s to $su(4)_f$ (Georgi and Glashow, 1974) received confirmation from the discovery of the ψ-boson, and the later discoveries of the charmed mesons.

Originally, the quark states were regarded simply as elements of the 3-dimensional FIR of $su(3)_f$. But with the success of the quark model in explaining many of the hadron properties and interactions, quarks were gradually assumed to be real spin-1/2 fermions. In order to satisfy the Pauli exclusion principle in states such as $\Delta^- = ddd$, quarks were further assumed to come in three different "colors."

In the meantime, the electroweak gauge field theory (Glashow, 1961; Weinberg, 1967; Salam, 1968) was developed, which established a linkage between the d and u quarks and the leptons e^- and ν_e. It was already known that the leptons included μ^- and ν_μ. With the later discovery of the leptons τ^- and ν_τ, and the evidence for hadrons containing charmed quarks c and bottom quarks b, the idea arose of three generations (families) of leptons and quarks,

$$(e^-, \nu_e; d, u), \quad (\mu^-, \nu_\mu; s, c), \quad (\tau^-, \nu_\tau; b, t) \tag{1.1}$$

The evidence that there are no more than three generations is obtained from the measurement of the Z^0 width, and in particular from the portion due to $\nu\bar{\nu}$ pairs. At present, no algebra has been proposed that yields a generation QN, and that limits the number of generations to three.

It will be seen in Section 5 that although it is fruitful to classify the six leptons into three generations, that is not the case for the six quarks. In fact, the assignment of quarks to three generation-doublets leads to multiplets that do not correspond to the observed hadron multiplets [see (5.3)], and all the benefits of the original $su(3)_f$, which led to the idea of quarks in the first place, would be lost. There is no doubt that at the very least, $su(3)_f$ brings a definite correct order to low-lying hadrons. All these benefits are retained by assuming that the quarks belong to a FIR of $su(6)_f$ which includes $su(3)_f$ and is a natural extension of it. This is precisely what is done in the lq-model of Section 5.

The success of the electroweak gauge field theory led to the development of quantum chromodynamics (QCD), also as a gauge field theory of strong interactions, in which color plays the same role as charge in quantum electrodynamics (QED).

In QCD one starts with massless "current" quarks that acquire mass via the Higgs mechanism. Since the estimated masses of the current quarks (e.g., 8 MeV for u and 16 MeV for d) are much smaller than the hadron masses, a hadron is expected to contain an infinite "sea" of $q\bar{q}$ pairs. The relationship between "current" quarks and the massive "constituent" quarks of the quark model that give the hadrons their QNs and properties is not clear. In particular, the hypothesis that a constituent quark is a "dressed" current quark is beset with difficulties (Keaton, 1994; Fritzsch, 1993). Moreover, there are many problems in understanding the hadron properties in terms of QCD dynamics (Raczka, 1993). There are no rules in QCD for how to make hadrons from quarks, and it is left up to the dynamics to determine what approximate symmetries should result. In the absence of a solution of the dynamical equations of QCD, it is desirable to formulate a model that does not depend upon quark dynamics, by considering the quark states to be elements of the FIR of an internal algebra.

We discuss three different versions of the model of the first type, based on three different sets of basic physical constituents: (1) The stable particles model in Section 2, (2) the lN-model in Section 3, and (3) the $l\Lambda'$-model in Section 4. Each succeeding version provides a more elaborate algebraic framework than the preceding ones, but they all predict *a priori* too many baryon states. It is possible that the dynamics of the physical constituents can lead to further restrictions on the number of compound baryon states. These models provide a simple intuitive meaning of the internal QNs and classify different types of "molecules" that one can build from the same constituents.

The only model of the second type that we discuss is the lq-model of Section 5, which is the main thrust of this paper. It is based on the internal algebra $su(4)_c \times su(6)_f$, where the subscript c stands for "color." The quark color-triplet is combined with a lepton color-singlet to form one FIR of $su(4)_c$. The $su(6)_f$ algebra is an extension of $su(3)_f$, and includes $su(4)_f$ and $su(5)_f$ quarks. To our knowledge, the analysis of the hadron multiplets of $su(5)_f$ and $su(6)_f$ is new. The $su(6)_f$ algebra should not be confused with the old $su(6)$ that contained $su(3)_f$ and $su(2)_{\text{spin}}$.

In this paper we concentrate on specifying all the ground-state hadron multiplets, with some discussion of the excited states. A full description of the excited states requires an extension of the internal dynamical algebra, perhaps to include $so(4, 2)$. It is then possible to calculate form factors (Barut, 1972, 1980a) and to explain the parton structure functions of hadrons without having to assume the physical reality of quarks.

2. THE STABLE PARTICLES MODEL

The basic idea of this model (Barut, 1980b) is that only the absolutely stable particles and their antiparticles can be considered as true building blocks of all other particles. In the first version of this model p, e^-, and ν_e and their antiparticles were taken to be the basic building blocks. One constructs from these first the states

$$n = pe^-\bar{\nu}_e, \qquad \nu_\mu = \nu_e(\nu_e\bar{\nu}_e)$$

which are the two next most stable particles, and since they are sufficiently long-lived, they can form the next level of rather stable states

$$\mu^- = \nu_\mu(e^-\bar{\nu}_e), \qquad \nu_\tau = \nu_\mu(\nu_e\bar{\nu}_e)$$

and the next level

$$\tau^- = \nu_\tau(e^-\bar{\nu}_e), \qquad \pi^- = \mu^-\bar{\nu}_\mu$$

and so on, with increasing complexity, higher mass, and hence decreasing stability. All mesons, baryons, and heavy leptons (even vector bosons) can be so constructed. The amount of flavors like strangeness, charm, etc., turns out to be naturally related to the number of semistable constituents μ^-, ν_μ, etc. They are conserved in the production, but due to their instability, they are not conserved when they decay, typically by a change of $\Delta S = 1$, which is the decay of the hadrons themselves.

One immediately asks whether light leptons can form very massive new states at all. This possibility has not been considered in the usual models, because one thinks a bound state should have a lower mass than the sum of the masses of the constituents. The clue is that *all* the states so formed are unstable resonances, and a deep potential well barrier with a high *positive* energy peak at very short distances is sufficient to produce very massive and sufficiently long-lived new resonance states. It has been shown in many models that magnetic and anomalous magnetic moment interactions of even relativistic Dirac particles can indeed produce such deep potential wells at short distances. At any rate, all we need is a structure in the short-distance interactions of leptons, without changing anything about their interactions at atomic distances, which magnetic interactions precisely seem to do.

Since the proton seems to be a composite object with an extended form factor, the model was further simplified to a truly leptonic model assuming only two absolutely stable constituents e^- and ν_e and their antiparticles (Barut, 1983). The quantum numbers of the proton are assumed to be the same as those of the $(e^+e^-e^+)$ system. The stability, the mass, and the magnetic moment of the proton must be understood dynamically.

In the stable particles model the exact calculation of the masses for two or more relativistic particles with magnetic interactions is a difficult problem, solved only in special cases or approximately. This is also the case in quark models. But then we have a clear intuitive picture for phenomena like *CP* violation, large-spin asymmetries, etc.

3. THE *lN*-MODEL

We call *flavor* the QN that distinguishes between each of the leptons $l = e^-, \mu^-, \tau^-$ and its neutrino ν_l on the one hand, and between the nucleons n and p on the other hand; and *generation* the QN that distinguishes between the three different l's or ν_l's. The simplest algebraic model of the first type whose physical constituents carry the flavor and generation QNs, as well as the QN that distinguishes between leptons and baryons, is the *lN*-model, whose basic constituents are those of Fig. 1 and their antiparticles.

It is instructive to explore this model to find out what hadron states it predicts and the problems it encounters. Moreover, according to the current ideas, the three lepton generations of flavor doublets

$$(e^-, \nu_e), \quad (\mu^-, \nu_\mu), \quad (\tau^-, \nu_\tau) \tag{3.1}$$

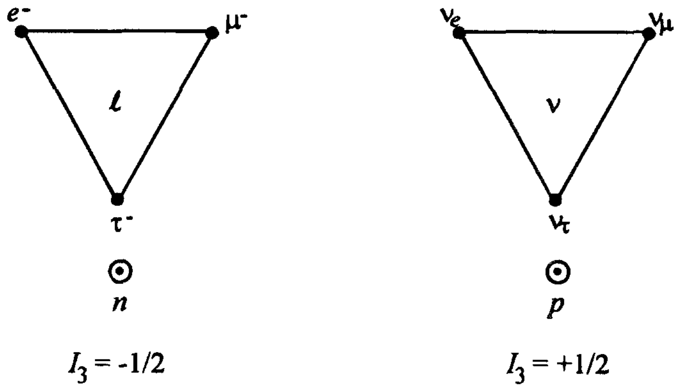


Fig. 1. *lN* fundamental multiplets.

are related to the three quark generations of flavor doublets

$$(d, u), \quad (s, c), \quad (b, t) \tag{3.2}$$

Thus many of the predictions of the *lN*-model can be applied to a quark model having the analogous structure.

A. The Model Algebra. According to the above discussion, the algebra whose FIR has the structure of Fig. 1 is

$$\mathcal{A}_{lN} = u(1)_{lN} \times su(2)_f \times su(3)_g \tag{3.3}$$

where f stands for "flavor" and g for "generation." The $u(1)_{lN}$ QN distinguishes between the leptons $su(3)_g$ triplet and the nucleons $su(3)_g$ singlet. The $su(2)_f$ algebra provides the flavor-isospin QNs that distinguish l from ν_l and n from p; and the $su(3)_g$ algebra the QNs that distinguish between the elements of the lepton triplets.

The basic physical constituents of the *lN*-model are the spin-1/2 fermions of Fig. 1 and their antiparticles, i.e.,

$$(e^-, \mu^-, \tau^-; n), \quad (\nu_e, \nu_\mu, \nu_\tau; p) \tag{3.4a}$$

$$(e^+, \mu^+, \tau^+; \bar{n}), \quad (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau; \bar{p}) \tag{3.4b}$$

Their states are described by the tensor product of Dirac 4-component spinors with the eigenstates of the FIR of $\mathcal{A}_{lN}$.

B. Meson Ground States. The meson ground states are obtained from the tensor product of the states of leptons and antileptons. With respect to $su(2)_f$, this gives

$$\underline{2} \otimes \underline{2} = \underline{3} \oplus \underline{1} \tag{3.5}$$

and with respect to $su(3)_g$, this gives

$$\underline{3} \otimes \underline{\bar{3}} = \underline{8} \oplus \underline{1} \tag{3.6}$$

Thus we have altogether $(3 + 1) \times (8 + 1) = 36$ meson ground states.

Since the leptons and antileptons are distinguishable from each other, the Pauli principle does not apply to their tensor products. Thus, both spins 0 and 1, resulting from the product of two spin-1/2 states, are allowed *a priori*. Consequently, we have 36 meson states with spin 0, and 36 with spin 1.

The flavor-isospin QNs of the multiplets on the RHS of (3.5) are given in Table I, where $I_3^{(f)}$ is also equal to the electric charge.

Each of these four flavor multiplets consists, according to (3.6), of a generation-octet plus a generation-singlet. For instance, the ground states of the *negatively* charged mesons consist of the octet

$$e^-\nu_\mu, \quad 2^{-1/2}(e^-\bar{\nu}_e - \mu^-\bar{\nu}_\mu), \quad \mu^-\bar{\nu}_e; \quad 2^{-1/2}(e^-\bar{\nu}_e + \mu^-\bar{\nu}_\mu) \tag{3.7a}$$

$$\tau^-\bar{\nu}_\mu, \quad \tau^-\bar{\nu}_e; \quad e^-\bar{\nu}_\tau, \quad \mu^-\bar{\nu}_\tau \tag{3.7b}$$

and the singlet

$$3^{-1/2}(e^-\bar{\nu}_e + \mu^-\bar{\nu}_\mu + \tau^-\bar{\nu}_\tau) \tag{3.7c}$$

Similarly, we have a positively charged $\underline{8} \oplus \underline{1}$, which gives the antimesons of (3.7), and two neutral $\underline{8} \oplus \underline{1}$ meson multiplets, as indicated in Table I.

To identify the states $\bar{l}l$ with the observed mesons, we make the following associations between the leptons and flavor QNs:

$$e^- \leftrightarrow d, \quad \nu_e \leftrightarrow u, \quad \mu^- \leftrightarrow s, \quad \nu_\mu \leftrightarrow c, \quad \tau^- \leftrightarrow b, \quad \nu_\tau \leftrightarrow t \tag{3.8}$$

Accordingly, we have, for example,

$$\pi^- = e^-\bar{\nu}_e, \qquad \pi^+ = \nu_e e^+, \qquad \pi^0 = 2^{-1/2}(e^-e^+ + \nu_e\bar{\nu}_e) \tag{3.9a}$$

$$K^- = \mu^-\bar{\nu}_e, \qquad K^+ = \nu_e\mu^+, \qquad K^0 = e^-\mu^+, \qquad \bar{K}^0 = \mu^-e^+ \tag{3.9b}$$

From (3.9) and (3.7) we see that the π meson triplet is scattered among three

Table I. Meson Flavor-Isospin Multiplets

$I^{(f)}$	1	1	1	0
$I_3^{(f)}$	-1	0	$+1$	0
Mesons	$l\bar{\nu}$	$2^{-1/2}(\bar{l}l + \nu\bar{\nu})$	$\nu\bar{l}$	$2^{-1/2}(\bar{l}l - \nu\bar{\nu})$

different octets, labeled by $I^{(f)} = +1$ and $I_3^{(f)} = -1, 0 + 1$. This feature is the same for the three-generations quark model.

C. Baryon Ground States. The baryon ground states are obtained from the formula

$$\text{baryon}(B) = \text{nucleon}(N) \otimes [\text{meson}(M)]^n \tag{3.10}$$

where $n = 0, 1, 2, 3 \ldots$. As examples of $n = 1, 2$, and 3, we have

$$\Sigma^- = nK^-, \qquad \Xi^- = (nK^0 + pK^-)K^- \tag{3.11a}$$

and

$$\Omega^- = (nK^0 + pK^-)K^0K^{*-} \tag{3.11b}$$

where K^{*-} has spin 1. This is essentially the old model of a baryon consisting of a nucleon with a meson cloud.

Again, since all particles in the tensor product (3.10) are distinguishable, all spins resulting from

$$\underline{2} \otimes (\underline{2} \otimes \underline{2}) = \underline{2} \otimes (\underline{1} \oplus \underline{3}) = \underline{2} \oplus (\underline{2} \oplus \underline{4}) \qquad \text{for} \quad n = 1 \tag{3.12}$$

are *a priori* allowed, i.e., each baryon state, for $n = 1$, occurs twice with spin 1/2 and once with spin 3/2.

The three main undesirable features of this model are: (1) In addition to the strange baryons (e.g., $N\mu^-e^+$) there are also unobservable antistrange baryon states (e.g., $Ne^-\mu^+$). (2) Since there is no limit on the exponent n in (3.10), there is an indefinite number of baryon ground states, each with too many spin possibilities. (3) The hadron multiplets do not correspond with the observed multiplets.

4. THE $l\Lambda'$-MODEL

This model is an extension of a model previously proposed by the authors (Basri and Barut, 1983) to include the (τ^-, ν_τ) leptons and the (bottom, top) flavors. Its main motivation is to obtain the observed hadron multiplets by using the three sextets, shown in Fig. 2, of leptons (l), antileptons ($\bar{l}'$),

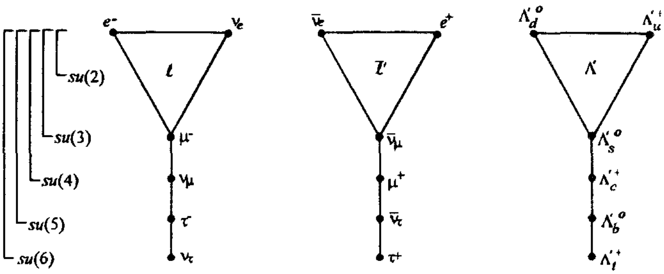


Fig. 2. $l\Lambda'$ fundamental multiplets.

and baryons (Λ') as the building blocks, instead of quarks (q). The prime on $\bar{l}'$ indicates that the antileptons ($\bar{l}$) are reordered to represent the FIR $\underline{6}$ of $su(6)_f \equiv su(6)_{\text{flavor}}$ instead of $\underline{\bar{6}}$; and the prime over Λ indicates that Λ' is a set of constituent (bare) baryons, and not the observed baryons. Each of these three sextets is a physical realization of the $\underline{6}$ FIR of $su(6)_f$.

The correspondence between these physical constituents and quarks is given in Table II. This correspondence is for the purpose of obtaining the correct QNs of the hadron states, and is not intended to be one-to-one between the basic fermions and quarks.

We assume that the hadron states are given by

$$\text{mesons} = l\bar{l}' \tag{4.1}$$

$$\text{baryons} = (l\bar{l}')\Lambda' \tag{4.2}$$

For example, $\pi^-(d\bar{u}) = e^-\bar{\nu}_e$, and the proton $p(du\,u)$ could be e^-e^+p', $\nu_e\bar{\nu}_e p'$, or $\nu_e e^+ n'$, depending on the dynamics.

One may ask why the mesons and baryons so formed have strong interactions. The strong interactions are short-ranged. When mesons or nucleons approach each other the constituent leptons will be subject to the short-distance magnetic interactions and, perhaps more importantly, the leptons will be exchanged. At large distances we have just the Coulomb forces and the tail of the magnetic interaction. We also remark that weak decays, which are also short-ranged, are due to the barrier penetration at short distances. For further discussion we refer to Barut (1980b).

Since all the fermions on the RHS of (4.1) and (4.2) are distinguishable, the Pauli principle does not apply to them, and thus all possible spins are allowed. Since l, $\bar{l}'$, Λ' all have spin 1/2, the number of hadron states in the $l\Lambda'$-model are

$$6^2 = 36 \quad \text{meson states with spins 0 and 1} \tag{4.3}$$

$$6^3 = 216 \quad \text{baryon states with spins 1/2, 1/2, and 3/2} \tag{4.4}$$

Table II. Correspondence Between l, $\bar{l}'$, Λ' and Quarks

	l	$\bar{l}'$	Λ'	Average charge
Flavor				
d	e^-	$\bar{\nu}_e$	$\Lambda_d'^0 \equiv n'$	$-1/3$
u	ν_e	e^+	$\Lambda_u'^+ \equiv p'$	$2/3$
s	μ^-	$\bar{\nu}_\mu$	$\Lambda_s'^0 \equiv \lambda'$	$-1/3$
c	ν_μ	μ^+	$\Lambda_c'^+$	$2/3$
b	τ^-	$\bar{\nu}_\tau$	$\Lambda_b'^0$	$-1/3$
t	ν_τ	τ^+	$\Lambda_t'^+$	$2/3$
Color	Green (G)	Red (R)	Blue (B)	

i.e., a total of $2 \times 36 = 72$ meson states, and $3 \times 216 = 648$ baryon states.

In order to be able to compare the $l\Lambda'$- and lq-models and get better insight into both, we present below all the $su(3)$ baryon multiplets that are obtained in the $l\Lambda'$-model.

Each of the three sextets in Fig. 2 consists of one $su(3)$ triplet $\underline{3}$ and three $su(3)$ singlets $\underline{1}$. Since (Halzen and Martin, 1984)

$$\underline{3} \otimes \underline{3} = \underline{6} \oplus \underline{\bar{3}}, \qquad \underline{6} \otimes \underline{3} = \underline{10}_S \oplus \underline{\bar{8}}_{S'}, \qquad \underline{\bar{3}} \otimes \underline{3} = \underline{8}_{A'} \oplus \underline{\bar{1}}_{A'} \tag{4.5}$$

where the subscript S means completely symmetric, A completely antisymmetric (a.s.), S' symmetric in the first two fermions, and A' a.s. in the first two fermions, then

$$(\underline{3} \otimes \underline{3}) \otimes \underline{3} = (\underline{10}_S \oplus \underline{\bar{8}}_{S'}) \oplus (\underline{8}_{A'} \oplus \underline{\bar{1}}_{A'}) \tag{4.6}$$

$$(\underline{3} \otimes \underline{3}) \otimes \underline{1} = (\underline{6} \oplus \underline{\bar{3}}_{S'}) \otimes \underline{1} = \underline{6} \oplus \underline{\bar{3}}, \qquad 3 \times 3 = 9 \text{ times} \tag{4.7}$$

since there are three different $\underline{3} \otimes \underline{3}$'s, and three different $\underline{1}$'s;

$$\underline{3} \otimes (\underline{1} \otimes \underline{1}) = \underline{3} \otimes \underline{1} = \underline{3}, \qquad 3 \times 3^2 = 27 \text{ times} \tag{4.8}$$

since there are three $\underline{3}$'s, and $3^2 = 9$ $(\underline{1} \otimes \underline{1})$'s;

$$\underline{1} \otimes \underline{1} \otimes \underline{1} = \underline{1}, \qquad 3^3 = 27 \text{ times} \tag{4.9}$$

This gives a total number of states equal to

$$(10 + 8 + 8 + 1) + (6 + 3) \times 9 + (3 \times 27) + 27 = 216 \tag{4.10}$$

as expected. The $su(3)$ multiplets content is

$$\underline{10}, \quad 2 \times \underline{8}, \quad \underline{1}_A, \quad 9 \times \underline{6}, \quad 9 \times \underline{\bar{3}}, \quad 27 \times \underline{3}, \quad 27 \times \underline{1} \tag{4.11}$$

This contains, for example, a spin-3/2 octet and a spin-1/2 decouplet which are not observed.

In the lq-model the meson states are given by (5.8) and the baryon states by (5.9). Since q and $\bar{q}$ are distinguishable, the Pauli principle does not apply to the meson states $q\bar{q}$, but it does apply to the baryon states qqq. Thus the meson states are the same in both the $l\Lambda'$- and lq-models. However, the baryon ground states are required in the lq-model to be completely antisymmetric with respect to the exchange of color, flavor, and spin of any two quarks. This severely limits the number of allowed baryon states to only 126, as shown in Table V, instead of the 648 predicted by the $l\Lambda'$-model.

If the underlying interactions at short distances have the proper symmetries, then perhaps one could understand the reduction of the number of possible baryon states from 648. For example, it is clear that the magnetic moment interactions are dependent on the spin orientation of the constituents, so that attractive forces can only occur for some spin orientations and not for others.

The internal algebra that gives the QNs specifying the multiplets of Fig. 2 is

$$\mathscr{A}_{l\Lambda'} = u(1)_{l\Lambda'} \times su(6)_f \tag{4.12}$$

The $u(1)_{l\Lambda'}$ QN distinguishes between l or $\bar{l}'$ and Λ' sextets, and the $su(6)_f$ QNs distinguish between the elements of each sextet.

The commuting elements of $su(6)_f$, whose eigenvalues provide the QNs specifying the individual states, are taken to be those of $su(6)_f$ and its subalgebras $su(N)_f$ for $N = 5, 4, 3, 2$, as shown in Fig. 2. This gives hadron multiplets that correspond to observed multiplets, but does not give the three generations of lepton flavor doublets (3.1).

The advantages of the $l\Lambda'$-model over the preceding ones in Sections 2 and 3 is that the total number of baryon ground states is finite, and there are no antistrange baryons, since there are no antiflavors in (4.2).

5. THE *lq*-MODEL

In the last three sections we discussed models of the first type. We now discuss a model of the second type, i.e., a model in which particle states are constructed from tensor products of spin eigenstates that belong to finite IRs of an internal algebra $\mathscr{A}$. These basic states, which can be characterized as "symmetry constituents," need not represent physical particles.

In the quark model, the masses of the current or constituent quarks that are assigned vary considerably from MeV to hundreds of GeV. It is difficult under these conditions to consider quarks to be identical fermions in order to apply the Pauli principle to them in the baryon states qqq. This problem does not arise if the quarks are considered to be symmetry constituents. They are then completely specified by the QNs of $\mathscr{A}$ and by spin, and are not assigned other properties such as mass and magnetic moment. These properties are observable only for the compound system, and not for its constituents.

A mass formula such as that of Gell–Mann and Okubo can be expressed purely in terms of the QNs describing the compound system, and the same is true for other physical properties. Even the parton structure of hadrons can be described by form factors that are function of the QNs of the compound system and its excited states (Barut, 1972, 1980a).

We regard experimental facts, about the $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ systems or any flavored hadrons as support for the internal algebra that yields the flavor QNs, rather than for the physical existence of a quark of particular flavor.

5.1. Basic Assumptions

(i) The leptons (l), quarks (q), and their antiparticles $\bar{l}$ and $\bar{q}$ are assumed to be spin-1/2 fermions whose states are described by Dirac 4-component spinors. Moreover, their states are also elements of the lowest-dimensional FIR of an internal algebra $\mathscr{A}$.

(ii) A tensor product of IRs of $\mathscr{A}$ must decompose into a direct sum of IRs of $\mathscr{A}$. This assumption excludes the inclusion of a Clifford algebra Cl_n in $\mathscr{A}$ since Cl_n has only one IR, and a tensor product of IRs of Cl_n would not lead to a direct sum of IRs of Cl_n.

(iii) We have

$$\mathscr{A} = su(4)_c \times su(6)_f \tag{5.1}$$

where the subscript c stands for "color" and f for "flavor."

The algebra $\mathscr{A}$ is not a symmetry algebra of the Hamiltonian, and thus masses of hadrons belonging to a multiplet can be very different from each other.

The $su(6)$ algebra has five FIRs described by the Young tableux designations

$$\underline{6} = \square = (10000), \qquad \underline{15} = \begin{smallmatrix}\square\\\square\end{smallmatrix} = (01000), \qquad \underline{20} = \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} = (00100)$$

$$\underline{\overline{15}} = \begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix} = (00010), \qquad \underline{\bar{6}} = \begin{smallmatrix}\square\\\square\\\square\\\square\\\square\end{smallmatrix} = (00001) \tag{5.2}$$

These IRs are fundamental in the sense that any IR of $su(6)$, $(\lambda_1, \lambda_2 \ldots \lambda_5)$, can be expressed as a linear combination of the five FIRs of (5.2).

However, all IRs of $su(6)$ can also be obtained from tensor products of $\underline{6}$ alone. For example, we have

$$\square \otimes \square = \begin{smallmatrix}\square\\\square\end{smallmatrix} \oplus \square\square, \qquad \begin{smallmatrix}\square\\\square\end{smallmatrix} \otimes \square = \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} \oplus \begin{smallmatrix}\square&\square\\\square&\end{smallmatrix}, \qquad \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} \otimes \square = \begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix} \oplus \begin{smallmatrix}\square&\square\\\square&\\\square&\end{smallmatrix}, \quad \text{etc.}$$

This is the reason why we represent the basic fermions by elements of $\underline{6}$ and not the other FIRs. The basic antifermions are represented by elements of $\underline{\bar{6}}$.

If instead of $su(6)_f$ in (5.1), we were to use the algebra

$$su(2)_{\text{flavor}} \times su(3)_{\text{generations}}$$

as in (3.3), to reflect the current ideas about three quark generations of flavor-doublets, then we would obtain hadron multiplets, as illustrated in (5.3), that have no correspondence to observed multiplets; and in particular, we would lose the meson octet and baryon octet and decouplet of the original $su(3)$ model that have had strong experimental support, and that were instrumental in the development of quark theory. For example if we rewrite (3.7*a*), (3.7*b*) with the help of the correspondences (3.8), we obtain the octet

$$D^- = d\bar{c}, \qquad \pi^- - D_s^- = d\bar{u} - s\bar{c}, \qquad K^- = s\bar{u}, \qquad \pi^- + D_s^- = d\bar{u} + s\bar{c} \tag{5.3a}$$

$$B_c^- = b\bar{c}, \qquad B^- = b\bar{u}, \qquad T^- = d\bar{t}, \qquad T_s^- = s\bar{t} \tag{5.3b}$$

(iv) We have

$$su(4)_c \supset su(3)_c \tag{5.4}$$

instead of $su(4)_c \supset su(2) \times su(2)$. Thus the FIR of $su(4)_c$, $\underline{4}_c$, consists of an $su(3)_c$ triplet $\underline{3}_c$, which is associated with quarks, and an $su(3)_c$ singlet $\underline{1}_c$, which is associated with leptons, as shown in Fig. 3. In this way, quarks and leptons are linked together as elements of a single multiplet, which is not the case in the standard $su(2) \times u(1)$ model (Glashow, 1961; Weinberg, 1967; Salam, 1968).

If $u(1)_{lq} \times su(3)_c$ were chosen instead of $su(4)_c$, then the leptons $\underline{1}_c$ would not be a FIR of $su(3)_c$, which violates assumption (ii).

(v) According to (5.1) and (iv), we have a lepton-$su(6)_f$ associated with $\underline{1}_c$, which we denote by $su(6)_l$, and three quark-$su(6)_f$'s associated with $\underline{3}_c$, which we denote collectively by $su(6)_q$. Since leptons do not interact strongly, whereas quarks do, the complete set of commuting operators of $su(6)_f$, whose QNs describe the l and q states, are different. This difference is expressed by the assumption

$$su(6)_l \supset su(2)_e \times su(2)_\mu \times su(2)_\tau \tag{5.5a}$$

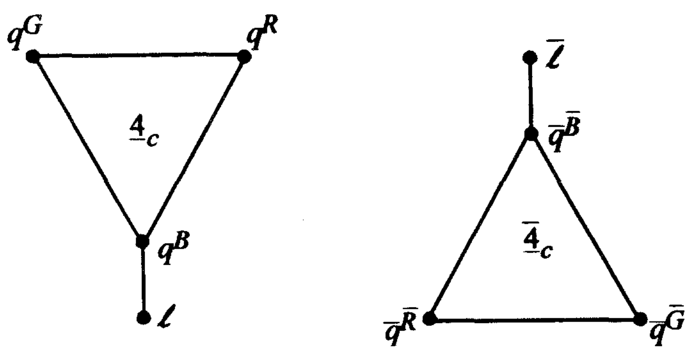


Fig. 3. The FIRs of $su(4)_c$.

and

$$su(6)_q \supset su(5)_q \supset su(4)_q \supset su(3)_q \supset su(2)_q \qquad (5.5b)$$

In accordance with this, the lepton states are described by the Casimir operators of $su(6)_l$, $su(2)_e$, $su(2)_\mu$, and $su(2)_\tau$, as well as $I^{(l)}$ and $I_3^{(l)}$ ($l = e, \mu, \tau$). The quark states, on the other hand, are described by the Casimir operators of $su(N)$, for $N = 2, \ldots, 6$, and all the other commuting operators of the different subalgebras, such as the hypercharge and isospin of $su(3)$.

A mass formula for the leptons or the hadrons formed from the quarks is a function of all these QNs. The same is true for all other hadron properties, such as magnetic moments, and no physical properties are assigned to quarks.

The FIRs $\underline{6}_l$, $\underline{\bar{6}}_{\bar{l}}$ of $su(6)_l$ and $\underline{6}_q$, $\underline{\bar{6}}_{\bar{q}}$ of $su(6)_q$ are shown in Figs. 4 and 5.

One can also decompose $\underline{6}_q$ into the two $su(3)$ triplets

$$(d, u; s), \quad (b, t; c)$$

However, the resulting hadron multiplets would not correspond to the observed multiplets.

(vi) Only color-singlets are observable. Thus leptons are observable, whereas quarks are not, since they belong to $\underline{3}_c$. From the tensor products

$$\underline{3} \otimes \underline{\bar{3}} = \underline{1} \oplus \underline{8} \qquad (5.6)$$

$$\underline{3} \otimes \underline{3} = \underline{\bar{3}} \oplus \underline{6} \qquad (5.7a)$$

$$(\underline{3} \otimes \underline{3}) \otimes \underline{3} = (\underline{\bar{3}} \oplus \underline{6}) \otimes \underline{3} = (\underline{1} \oplus \underline{8}) \oplus (\underline{8}' \oplus \underline{10}) \qquad (5.7b)$$

we see that $q\bar{q}$ and qqq are observable, since they have a singlet term in (5.6) and (5.7), but qq is not observable.

(vii) In accordance with (5.6) and (5.7), we assume that the hadron states are given by the color singlets

$$\text{meson}(q_a\bar{q}_b) = 3^{-1/2} \sum_k q_a^{(k)}\bar{q}_b^{(\bar{k})} \qquad (5.8)$$

$$\text{baryon}(q_a q_b q_c) = 6^{-1/2}\, \epsilon_{jkl} q_a^{(j)} q_b^{(k)} q_c^{(l)} \qquad (5.9)$$

where $a, b, c = 1, \ldots, 6$ range over the six flavors of $\underline{6}_q$, and j, k, l range over the three colors G, R, B of $\underline{3}_c$; ϵ_{jkl} is the Levi-Civita symbol.

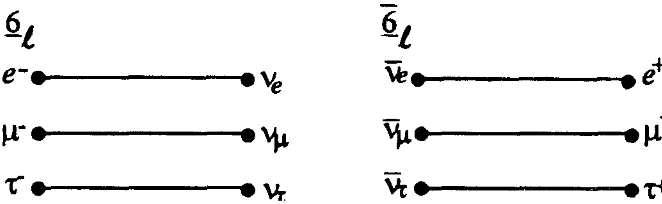


Fig. 4. The FIRs of $su(6)_l$.

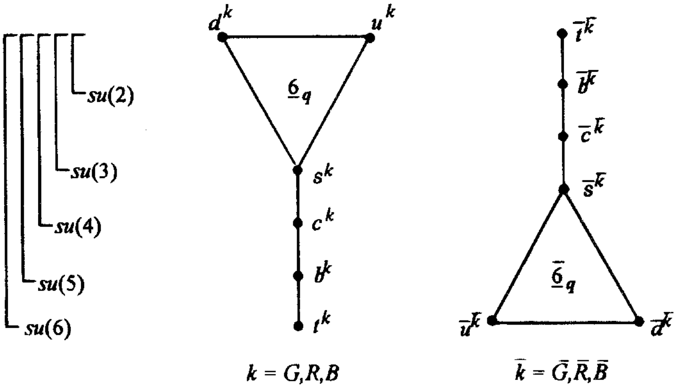


Fig. 5. The FIRs of $su(6)_q$.

These formulas do not necessarily imply that hadrons are compound systems consisting of real constituent quarks. They are useful in predicting hadron state multiplets; and it is not understood why they work as well as they do.

If quarks were assumed to be real, then it should be noted that the "constituent" quarks $q_a^{(k)}$ involved in (5.8) and (5.9) are not the "current" quarks of QCD.

If (5.6) is applied to quarks, then the color-singlet $\underline{1}_c$ of (5.8) is identified with mesons, and the color-octet $\underline{8}_c$ with the eight gluons

$$G\bar{R},\ G\bar{B},\ R\bar{G},\ R\bar{B},\ B\bar{G},\ B\bar{R},\ G\bar{G} - R\bar{R},\ G\bar{G} + R\bar{R} - 2B\bar{B} \qquad (5.10)$$

Since a gluon does not belong to $\underline{1}_c$, it is not observable. But two gluons could form a color-singlet (glueball), which could be observable.

(viii) The baryon ground states qqq are totally a.s. with respect to the interchange of the color, flavor, and spin of any two quarks. Note that the spin comes from the Dirac algebra.

Since, according to (5.9), the baryon states are color-a.s., this implies that the baryon ground states must be totally flavor-spin symmetric. This conclusion plays a crucial role in the construction of the baryon ground states in Section 5.4.

If quarks were real, then all orbital angular momenta of qqq would be zero in the ground state, and a baryon ground state would be totally space symmetric. When (viii) is combined with this, it would be equivalent to the Pauli principle applied to the spin-1/2 identical fermions q. In baryon excited states the orbital angular momenta are not zero, and the space symmetry changes together with the symmetry of the rest of the state.

If quarks were not real, then it is not meaningful to assign them space coordinates. However, it is still possible to assign an intrinsic parity η to the whole baryon rest state. The sign of η for any baryon state is arbitrary, since baryons are fermions, but it should be possible to fix the sign of η/η_0, where η_0 refers to the ground state, by the dynamical algebra describing the baryon excited states (Barut, 1972, 1980*a*). We can then make the following assumption regarding the symmetry of the baryon excited states:

(ix) A baryon excited state $(qqq)'$ is totally a.s. with respect to the interchange of color, flavor, and spin coordinates of any two quarks if $\eta/\eta_0 = +1$, and totally symmetric if $\eta/\eta_0 = -1$.

This, in effect, restates the Pauli principle by replacing symmetry with respect to exchange of space coordinates by η/η_0. For real quarks, these two quantities are related by $\eta/\eta_0 = (-1)^L$. Assumption (ix) allows us to apply the Pauli principle without committing ourselves to the reality of quarks, and simply treating quark states as eigenstates of an internal algebra, in which case the total orbital angular momentum L has no meaning.

It follows from (ix) that there are two sets of baryon multiplets, one with $\eta/\eta_0 = +1$, and the other with $\eta/\eta_0 = -1$. Each set is the base of a tower of excited baryon states with successive states differing from each other by one unit of total angular momentum. This is a definite prediction that can be tested experimentally.

Since q and $\bar{q}$ are distinguishable by the *class* (Gourdin, 1967) (see Section 5.2) of the IR to which they belong, the Pauli principle, or (ix), does not apply to the meson $q\bar{q}$ states. Thus meson excited states form one tower over the ground states with successive states separated by one unit of angular momentum, and alternating in the sign of η (Barut, 1972, 1980*a*).

(x) The lowest-dimensional inequivalent FIRs of $su(4)_c$ are $\underline{4}_c$ and $\bar{\underline{4}}_c$, and of $su(6)_f$ are $\underline{6}_f$ and $\bar{\underline{6}}_f$. We assume that the only FIRs of $\mathscr{A}$ that are of physical significance are

$$\underline{4}_c \otimes \underline{6}_f \qquad \text{and} \qquad \bar{\underline{4}}_c \otimes \bar{\underline{6}}_f \tag{5.11}$$

i.e., we exclude the possibilities $\underline{4}_c \otimes \bar{\underline{6}}_f$ and $\bar{\underline{4}}_c \otimes \underline{6}_f$.

We identify $\underline{4}_c \otimes \underline{6}_f$ with the basic fermions (l, q), and $\bar{\underline{4}}_c \otimes \bar{\underline{6}}_f$ with the basic antifermions $(\bar{l}, \bar{q})$. Thus the antifermions have both antiflavor and anticolor. The eight sextets of (5.11) are shown in Figs. 4 and 5.

There is one more assumption concerning vector bosons, which will be stated at the end of Section 5.2.

5.2. Quantum Numbers of Leptons and Quarks

According to (5.5a), we decompose $su(6)_l$ into three disjoint isospin $su(2)$ subalgebras, which we denote by

$$su(2)_e, \quad su(2)_\mu, \quad su(2)_\tau \tag{5.12}$$

Table III. Leptons Quantum Numbers

	e^-, ν_e	μ^-, ν_μ	τ^-, ν_τ
L_e	1, 1	0, 0	0, 0
L_μ	0, 0	1, 1	0, 0
I_{e3}	$-\frac{1}{2}, \frac{1}{2}$	0, 0	0, 0
$I_{\mu 3}$	0, 0	$-\frac{1}{2}, \frac{1}{2}$	0, 0
$I_{\tau 3}$	0, 0	0, 0	$-\frac{1}{2}, \frac{1}{2}$

where the subscripts e, μ, τ refer to the electron, muon, and taon, respectively. Each subalgebra contains an isospin component $\hat{I}_3$, whose eigenvalues are shown in Table III.

The algebra $su(6)$ has altogether five mutually commuting operators. For $su(6)_l$, we choose these to be

$$\hat{L}_e, \hat{L}_\mu, \hat{I}_3^e, \hat{I}_3^\mu, \hat{I}_3^\tau \tag{5.13}$$

The eigenvalues L_e, L_μ of $\hat{L}_e$, $\hat{L}_\mu$, shown in Table III, serve to distinguish the three $su(2)$ subalgebras (5.12) from each other. The QNs of the antileptons $\bar{l}$ are opposite in sign to those of the leptons l in Table III.

We define for $su(6)_l$ the lepton-isospin component

$$I_3^l \equiv I_3^e + I_3^\mu + I_3^\tau \tag{5.14}$$

Similarly, $su(6)_q$ has five mutually commuting elements $\hat{H}_a$ ($a = 1, \ldots 5$), whose eigenvalues are given in Table IV. The eigenvalue H_1 distinguishes between the element of $su(2)_q$, H_2 between the $\underline{2}$ and $\underline{1}$ $su(2)_q$ IRs of $su(3)_q$, H_3 between the $\underline{3}$ and $\underline{1}$ $su(3)_q$ IRs of $su(4)_q$, H_4 between the $\underline{4}$ and $\underline{1}$ $su(4)_q$ IRs of $su(5)_q$, and H_6 between the $\underline{5}$ and $\underline{1}$ $su(5)_q$ IRs of $su(6)_q$. The antiquarks $\bar{q}$ have QNs that are opposite in sign to those in Table IV.

For $su(6)_q$ we define a quark-isospin component by

$$I_3^q = H_1 + (\tfrac{1}{2}H_2 - \tfrac{2}{3}H_3) + (\tfrac{1}{2}H_4 - \tfrac{3}{5}H_5) \tag{5.15}$$

Table IV. Quarks Quantum Numbers

	d	u	s	c	b	t
H_1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
H_2	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	0	0
H_3	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
H_4	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$-\frac{4}{5}$	0
H_5	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{5}{6}$

The FIRs $\underline{6}_f$ and $\underline{\bar{6}}_f$ of $su(6)_f$ are distinguished from each other by a *class* QN (Gourdin, 1967), which is a function of the Casimir operators of $su(6)_f$, and is a linear function of the numbers specifying the Young tableau of the FIRs. For $su(6)_l$, this QN is identified with

$$\text{lepton number } (L) = \begin{cases} +1 & \text{for leptons} \\ -1 & \text{for antileptons} \end{cases} \tag{5.16}$$

and for $su(6)_q$ with

$$\text{baryon number } (B) = \begin{cases} +1/3 & \text{for quarks} \\ -1/3 & \text{for antiquarks} \end{cases} \tag{5.17}$$

The class QN for the FIRs (5.2) has the respective values

$$L = 1, 2, 3, -2, -1 \qquad \text{for } su(6)_l$$

and

$$B = 1/3, 2/3, 1, -2/3, -1/3 \qquad \text{for } su(6)_q$$

If we assume that only hadron states with integer values of B are observable, then only the $\underline{20}$ FIR is observable. This is indeed one of the IRs resulting from the tensor product $qqq = \underline{6} \otimes \underline{6} \otimes \underline{6}$, as shown in (5.45). Since

$$B(q\bar{q}) = 0, \qquad B(qqq) = +1, \qquad B(\bar{q}\,\bar{q}\,\bar{q}) = -1$$

then $q\bar{q}$, qqq and their antiparticles are all observable.

Although the QNs L and B do distinguish between the basic fermions and antifermions, they cannot replace the particle–antiparticle QN, A. For example, π^- and π^+ both have $B = 0$, yet they are different and form a particle–antiparticle pair.

The QN A is equal to the combined operation CPT, where C is charge conjugation, P is the parity, and T is the time reversal. The operations P and T refer to Poincaré states in general, and thus to both bosons and fermions, and not just to Dirac fermion states. Since C, P, and T are all multiplicative, so is $A = CPT$. Thus A is conserved multiplicatively, whereas L and B are conserved additively.

With the help of (5.16) we can define

$$\text{tauon number } (L_\tau) \equiv L - L_e - L_\mu \tag{5.18}$$

which has the values $+1$ for τ^-, ν_τ and -1 for τ^+, $\bar{\nu}_\tau$.

It was stated below (5.4) that the FIR $\underline{4}_c$ of $su(4)_c$ consists of the IRs $\underline{3}_c$ and $\underline{1}_c$ of $su(3)_c$. These IRs are distinguished from each other by the QN

$$\text{color hypercharge } (Y_c) = \begin{cases} -1 & \text{for leptons} \\ +1/3 & \text{for quarks} \end{cases} \tag{5.19}$$

with opposite signs for the antifermions. Note that Y_c is one of the three QNs that specify the elements of an IR of $su(4)_c$. The other two QNs specify the elements of an IR of $su(3)_c \subset su(4)_c$.

The electric charges of leptons and quarks are given by

$$Q_l = \frac{1}{2} Y_c + I_3^l, \qquad Q_q = \frac{1}{2} Y_c + I_3^q \tag{5.20}$$

and have the values

$$Q(e^-, \mu^-, \tau^-) = -1, \; Q(\nu_e, \nu_\mu, \nu_\tau) \quad = 0 \tag{5.21a}$$

$$Q(d, s, t) = -1/3, \qquad Q(u, c, t) = +2/3 \tag{5.21b}$$

as can be verified from (5.19), (5.15), (5.14), and Tables III and IV. The charges (5.20) are related to each other by

$$Q_q = Q_l + \frac{2}{3} \tag{5.22}$$

We are now in a position to state the final assumption of the lq-model.

(xi) The vector boson states are given by the expressions

$$\text{photon } (\gamma) = 2^{-1/2} \sum_l (l^- l^+ - \nu_l \bar{\nu}_l) \tag{5.23a}$$

$$W^- = \sum_l l^- \bar{\nu}_l, \qquad Z^0 = 2^{-1/2} \sum_l (l^- l^+ + \nu_l \bar{\nu}_l), \qquad W^+ = \sum_l \nu_l l^+ \tag{5.23b}$$

where the sums are over $l = e, \mu, \tau$. Note that in (5.23)

$$L_e = 0, \qquad L_\mu = 0, \qquad L_\tau = 0 \tag{5.24}$$

and thus there is no mixing between the elements of the three $su(2)$ subalgebras (5.12). In effect, this assumes the conservation of L_e, L_μ, and L_τ. If neutrino oscillations ($\nu_\tau \to \nu_\mu \to \nu_e$) is established experimentally, then terms of the form $e^- \bar{\nu}_\mu$, etc., would need to be added to (5.23).

An $su(2)$ algebra has one FIR, which is $\underline{2}$, and

$$\underline{2} \otimes \underline{2} = \underline{1} \oplus \underline{3} \tag{5.25}$$

The photon state (5.23a) is an element of $\underline{1}$, and the vector boson triplet (5.23b) constitutes $\underline{3}$.

Since l and $\bar{l}$ are distinguishable, $l\bar{l}$ can occur with spin 1 or 0. All the states (5.23) occur only with spin 1. This may have something to do with the nature of electroweak interactions, and is a fact that needs explanation.

5.3. Meson Ground States

The meson ground states are constructed from the color-singlet (5.8). In accordance with the q-sextet of Fig. 5 and the decomposition (5.5b), we give below the membership of every meson multiplet to the IR of $su(6)_q$ and all its subalgebras, $su(5)_q$, $su(4)_q$, $su(3)_q$, and $su(2)_q$. For this purpose we introduce the following definitions:

$(p)_N$ denotes the p-dimensional IR of $su(N)$.

$(p)_N = (q, r)_{N-1}$ denotes the decomposition of $(p)_N$ into the q- and r-IRs of the subalgebra $su(N - 1)$.

$(p \otimes q)_N$ denotes the tensor product of p- and q-dimensional IRs of $su(N)$.

Starting with $su(6)$, the meson states are given by

$$(6 \otimes \bar{6})_6 = (1, 5)_5 \otimes (\bar{1}, \bar{5})_5 = (\bar{1})_6 \oplus (35)_6 \tag{5.26a}$$

$$(\bar{1})_6 = (1 \otimes \bar{1})_5, \qquad (35)_6 = (1 \otimes \bar{5})_5 \oplus (5 \otimes \bar{1})_5 \oplus (5 \otimes \bar{5})_5 \tag{5.26b}$$

$$(5 \otimes \bar{5})_5 = (1, 4)_4 \otimes (\bar{1}, \bar{4})_4 = (\bar{1})_5 \oplus (24)_5 \tag{5.27a}$$

$$(\bar{1})_5 = (1 \otimes \bar{1})_4, \qquad (24)_5 = (1 \otimes \bar{4})_4 \oplus (4 \otimes \bar{1})_4 \oplus (4 \otimes \bar{4})_4 \tag{5.27b}$$

$$(4 \otimes \bar{4})_4 = (1, 3)_3 \otimes (\bar{1}, \bar{3})_3 = (\bar{1})_4 \oplus (15)_4 \tag{5.28a}$$

$$(\bar{1})_4 = (1 \otimes \bar{1})_3, \qquad (15)_4 = (1 \otimes \bar{3})_3 \oplus (3 \otimes \bar{1})_3 \oplus (3 \otimes \bar{3})_3 \tag{5.28b}$$

$$(3 \otimes \bar{3})_3 = (1, 2)_2 \otimes (\bar{1}, \bar{2})_2 = (\bar{1})_3 \oplus (8)_3 \tag{5.29a}$$

$$(\bar{1})_3 = (1 \otimes \bar{1})_2, \qquad (8)_3 = (1 \otimes \bar{2})_2 \oplus (2 \otimes \bar{1})_2 \oplus (2 \otimes \bar{2})_2 \tag{5.29b}$$

$$(2 \otimes \bar{2})_2 = (\bar{1})_2 \oplus (3)_2 \tag{5.30}$$

For instance, $(3)_2 \in (8)_3 \in (15)_4 \in (24)_5 \in (35)_6$.

In this way, the membership of every isospin $su(2)$ multiplet can be traced to its ancestor $su(3)$, $su(4)$, $su(5)$, and $su(6)$ multiplets.

All the meson multiplets resulting from (5.26) to (5.30) are presented in Fig. 6. Since q and $\bar{q}$ are distinguishable, each of the 36 meson states occurs with both spin 0 (pseudoscalar) and spin 1 (vector). The meson symbols in Fig. 6 are those of the pseudoscalar mesons.

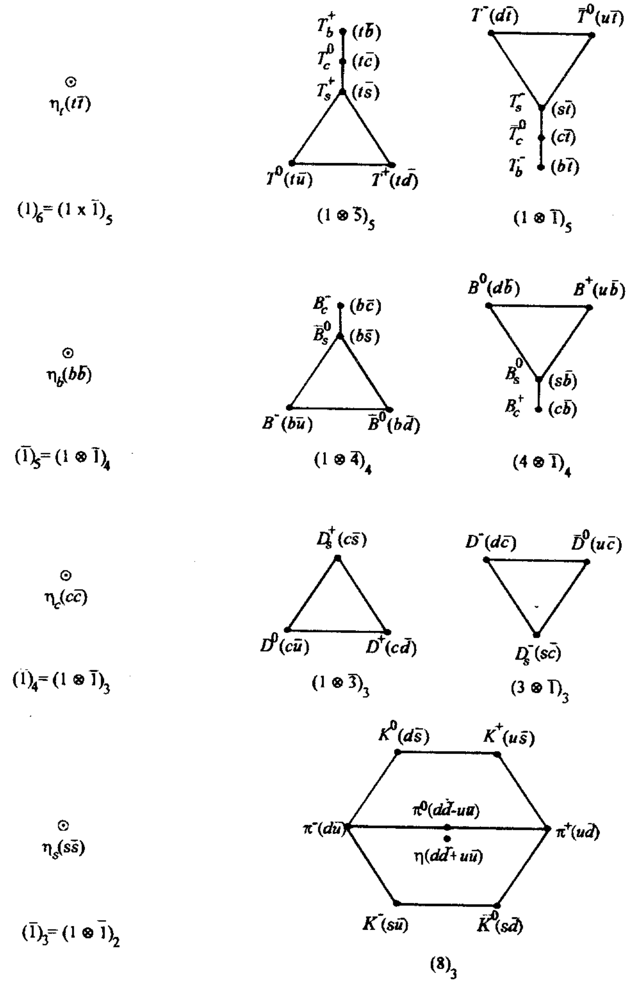


Fig. 6. Pseudoscalar meson multiplets.

5.4. Baryon Ground States

The baryon ground states are constructed from the color-singlet states (5.9). It was pointed out in Section 5.1 in the discussion of assumption (viii) that the flavor-spin factor of a baryon state must be completely symmetric. For this reason, instead of proceeding as in (5.26)–(5.30), we first examine the symmetry of the spin states and then find the appropriate flavor states associated with them. This will entail combining several state representations of the same IR to obtain flavor states of the correct symmetry.

The spin states obtained from (3.12) are

$$|3/2\rangle_{3/2} = (+\ +\ +), \qquad |3/2\rangle_{-3/2} = (-\ -\ -) \tag{5.31a}$$

$$|3/2\rangle_{1/2} = 3^{-1/2}[(+\ +\ -) + (+\ -\ +) + (-\ +\ +)] \tag{5.31b}$$

$$|3/2\rangle_{-1/2} = 3^{-1/2}[(-\ -\ +) + (-\ +\ -) + (+\ -\ -)] \tag{5.31c}$$

$$|1/2(S_{12})\rangle_{1/2} = 6^{-1/2}[(+\ -\ +) + (-\ +\ +) - 2(+\ +\ -)] \tag{5.32a}$$

$$|1/2(S_{12})\rangle_{-1/2} = 6^{-1/2}[(-\ +\ -) + (+\ -\ -) - 2(-\ -\ +)] \tag{5.32b}$$

$$|1/2(S_{13})\rangle_{1/2} = 6^{-1/2}[(+\ +\ -) + (-\ +\ +) - 2(+\ -\ +)] \tag{5.32c}$$

$$|1/2(S_{13})\rangle_{-1/2} = 6^{-1/2}[(-\ -\ +) + (+\ -\ -) - 2(-\ +\ -)] \tag{5.32d}$$

$$|1/2(S_{23})\rangle_{1/2} = 6^{-1/2}[(+\ +\ -) + (+\ -\ +) - 2(-\ +\ +)] \tag{5.32e}$$

$$|1/2(S_{23})\rangle_{-1/2} = 6^{-1/2}[(-\ -\ +) + (-\ +\ -) - 2(+\ -\ -)] \tag{5.32f}$$

$$|1/2(A_{12})\rangle_{1/2} = 2^{-1/2}[(+\ -\ +) - (-\ +\ +)] \tag{5.33a}$$

$$|1/2(A_{12})\rangle_{-1/2} = 2^{-1/2}[(-\ +\ -) - (+\ -\ -)] \tag{5.33b}$$

$$|1/2(A_{13})\rangle_{1/2} = 2^{-1/2}[(+\ +\ -) - (-\ +\ +)] \tag{5.33c}$$

$$|1/2(A_{13})\rangle_{-1/2} = 2^{-1/2}[(-\ -\ +) - (+\ -\ -)] \tag{5.33d}$$

$$|1/2(A_{23})\rangle_{1/2} = 2^{-1/2}[(+\ +\ -) - (+\ -\ +)] \tag{5.33e}$$

$$|1/2(A_{23})\rangle_{-1/2} = 2^{-1/2}[(-\ -\ +) - (-\ +\ -)] \tag{5.33f}$$

The index S_{ij} (A_{ij}) means symmetric (antisymmetric) with respect to interchange of the spins in the ith and jth places.

To construct the baryon multiplets, we proceed as in (4.6)–(4.11) with the *condition* that the flavor-spin states must be completely symmetric. For the multiplets (4.6) resulting from $\underline{3} \otimes \underline{3} \otimes \underline{3}$, the only possibilities are (Halzen and Martin, 1984)

$$\underline{10}\,|3/2\rangle, \qquad \sum_{i<j} [\underline{8}(S_{ij})\,|1/2(S_{ij})\rangle + \underline{8}(A_{ij})\,|1/2(A_{ij})\rangle] \tag{5.34}$$

The totally a.s. IR $\underline{1}_A$ does not contribute to the ground state because there is no spin state it can be combined with to form a totally symmetric flavor-spin state. But it could contribute to an excited state with an a.s. space wavefunction (Halzen and Martin, 1984). Thus we obtain only $10 + 8 = 18$ states from $\underline{3} \otimes \underline{3} \otimes \underline{3}$, compared to the $27 \times 3 = 81$ states given by (4.6).

For the tensor product of two $\underline{3}$'s and one $\underline{1}$, we have the three possible combinations

$$(\underline{3} \otimes \underline{3}) \otimes \underline{1} = \underline{6}(S_{12}) \oplus \underline{\bar{3}}(A_{12}) \tag{5.35a}$$

$$(\underline{3} \otimes \underline{1}) \otimes \underline{3} = \underline{6}(S_{13}) \oplus \underline{\bar{3}}(A_{13}), \qquad \underline{1} \otimes (\underline{3} \otimes \underline{3}) = \underline{6}(S_{23}) \oplus \underline{\bar{3}}(A_{23}) \tag{5.35b}$$

There are two different ways of obtaining completely symmetric flavor-spin states for $\underline{6}$, namely

$$\underline{6}(1/2) \equiv \sum_{i<j} \underline{6}(S_{ij})|1/2(S_{ij})\rangle \tag{5.36a}$$

$$\underline{6}(3/2) \equiv [\underline{6}(S_{12}) + \underline{6}(S_{13}) + \underline{6}(S_{23}))]|3/2\rangle \tag{5.36b}$$

but only one way for $\underline{\bar{3}}$, i.e.,

$$\underline{\bar{3}}(1/2) \equiv \sum_{i<j} \underline{\bar{3}}(A_{ij})|1/2(A_{ij})\rangle \tag{5.37}$$

Thus $\underline{6}$ occurs with both spins 3/2 and 1/2, whereas $\underline{\bar{3}}$ can only occur with spin 1/2.

In the case $\underline{3} \otimes \underline{1} \otimes \underline{1}$, where $\underline{3}$ is the triplet (d, u; s) and $\underline{1}$ is c, b, or t, a completely symmetric flavor-spin state can be formed, once with spin 3/2 and twice with spin 1/2. If $p \in \underline{3}$ and $x, y \in \underline{1}$, then we have

$$\underline{3}(pxy)_{3/2} \equiv (pxy + xyp + ypx + yxp + xpy + pyx)|3/2\rangle \tag{5.38}$$

$$\underline{3}(pxy)_{1/2} \equiv (pxy + xpy)|1/2(S_{12})\rangle + (pyx + xyp)|1/2(S_{13})\rangle + (ypx + yxp)|1/2(S_{23})\rangle \tag{5.39a}$$

$$\underline{3}[pxy]_{1/2} \equiv (pxy - xpy)|1/2(A_{12})\rangle + (pyx - xyp)|1/2(A_{13})\rangle + (ypx - yxp)|1/2(A_{23})\rangle \tag{5.39b}$$

However, when $y = x$, we find that

$$\underline{3}(pxx)_{1/2} = \underline{3}[pxx]_{1/2} \tag{5.40}$$

and spin 1/2 occurs only once.

Finally, for the case $\underline{1} \otimes \underline{1} \otimes \underline{1}$, completely symmetric singlet flavor-spin states

$$\underline{1}(xyz)_{3/2}, \quad \underline{1}(xyz)_{1/2}, \quad \underline{1}[xyz]_{1/2} \tag{5.41}$$

are obtained exactly as in (5.38) and (5.39). As in (5.40),

$$\underline{1}(xyy)_{1/2} = \underline{1}[xyy]_{1/2} \tag{5.42}$$

and spin 1/2 occurs only once. Furthermore, when $x = y = z$, only

$$\underline{1}(xxx)_{3/2} \tag{5.43}$$

is possible. Thus (xxx) occurs only with spin 3/2, (xyy) occurs once with spin 3/2 and once with spin 1/2, and (xyz) occurs once with spin 3/2 and twice with spin 1/2.

Note that there are three states of type (xxx),

$$ccc, \quad bbb, \quad ttt \tag{5.44a}$$

six states of type (xyy),

$$(ccb), \quad (cct), \quad (cbb), \quad (bbt), \quad (ctt), \quad (btt) \tag{5.44b}$$

and one state of type (xyz), namely (cbt).

According to the above analysis, the numbers of baryon states at the $su(4)$, $su(5)$, and $su(6)$ levels are as given in Table V. For $su(6)$ we have 70 baryons with spin 1/2 and 56 with spin 3/2.

This is in agreement with the decomposition

$$(\underline{6} \otimes \underline{6}) \otimes \underline{6} = (\underline{15} \oplus \underline{21}) \otimes \underline{6} = (\underline{20}_A \oplus \underline{70}) \oplus (\underline{70} \oplus \underline{56}_S) \tag{5.45}$$

The totally a.s. IR $\underline{20}_A$ does not contribute to the ground state for the same reason that the $\underline{1}_A$ resulting from $\underline{3} \otimes \underline{3} \otimes \underline{3}$ was excluded. The 20 a.s. states are obtained as follows: one from $\underline{3} \otimes \underline{3} \otimes \underline{3}$, nine from $\underline{3} \otimes \underline{3} \otimes \underline{1}$, nine from $\underline{3} \otimes \underline{1} \otimes \underline{1}$, and one from $\underline{1} \otimes \underline{1} \otimes \underline{1}$.

The two $\underline{70}$ IRs are symmetric or a.s. with respect to the interchange of two baryons, and can be combined together with the spin-1/2 states (5.32)

Table V. Number of Baryon Ground States

		$su(4)$		$su(5)$		$su(6)$	
	Spin-	1/2	3/2	1/2	3/2	1/2	3/2
$\underline{3} \otimes \underline{3} \otimes \underline{3}$	$su(3)$	8	10	$1 \times 8 = 8$	$1 \times 10 = 10$	$1 \times 8 = 8$	$1 \times 10 = 10$
$\underline{3} \otimes \underline{3} \otimes \underline{1}$	$\underline{6}$	6	6	$2 \times 6 = 12$	$2 \times 6 = 12$	$3 \times 6 = 18$	$3 \times 6 = 18$
	$\underline{3}$	3	0	$2 \times 3 = 6$	0	$3 \times 3 = 9$	0
$\underline{3} \otimes \underline{1} \otimes \underline{1}$	$\underline{3}$ (pxx)	3	3	$2 \times 3 = 6$	$2 \times 3 = 6$	$3 \times 3 = 9$	$3 \times 3 = 9$
	$\underline{3}$ (pxy)	0	0	$3 + 3 = 6$	$1 \times 3 = 3$	$3(3 + 3) = 18$	$3 \times 3 = 9$
$\underline{1} \otimes \underline{1} \otimes \underline{1}$	$\underline{1}$ (xxx)	0	1	0	$2 \times 1 = 2$	0	$3 \times 1 = 3$
	$\underline{1}$ (xyy)	0	0	$2 \times 1 = 2$	$2 \times 1 = 2$	$6 \times 1 = 6$	$6 \times 1 = 6$
	$\underline{1}$ (xyz)	0	0	0	0	$1 + 1 = 2$	$1 \times 1 = 1$
		20	20	40	35	70	56
		40		75		126	

and (5.33) to obtain totally symmetric flavor-spin states, as was done in (5.34) with the two $\underline{8}$ IRs.

The totally symmetric states of $\underline{56}_S$ combine with the spin-3/2 states (5.31) to give the 56 spin-3/2 baryon states.

To our knowledge, the analysis of the baryon multiplets of $su(5)_f$ and $su(6)_f$ is new.

All the baryon multiplets are shown in Fig. 7. They include the $\underline{8}(1/2)$ and $\underline{10}(3/2)$, which have considerable experimental support. These multiplets are not predicted by the quark (three-generations) model. Very few of the multiplets that include the c, b, or t quarks have been observed (Review of Particle Properties, 1992). Figure 7 should provide a useful guide to the prediction and classification of new baryon ground states.

5.5. Baryon Excited-State Multiplets with $\eta/\eta_0 = -1$

To obtain these multiplets we proceed as in Section 5.4, with the difference that now the flavor-spin state must be totally a.s.

For

$$\underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1}_A + \underline{8}_{S'} + \underline{8}_{A'} + \underline{10}_S \tag{5.46}$$

we obtain the two a.s. multiplets

$$\underline{1}(3/2) = 6^{-1/2}(dus + usd + sdu - sud - uds - dsu)|3/2\rangle \tag{5.47}$$

$$\underline{8}(1/2) = \sum_{i<j} [\underline{8}(S_{ij})|1/2(A_{ij})\rangle + \underline{8}(A_{ij})|1/2(S_{ij})\rangle] \tag{5.48}$$

For

$$\underline{3} \otimes \underline{3} \otimes \underline{1} = (\underline{\bar{3}}_{A'} \oplus \underline{6}_{S'}) \otimes \underline{1} \tag{5.49}$$

we obtain for each c, b, $t \in \underline{1}$, the a.s. multiplets

$$\underline{\bar{6}}(1/2) = \sum_{i<j} \underline{\bar{6}}(S_{ij})|1/2(A_{ij})\rangle \tag{5.50}$$

$$\underline{\bar{3}}(1/2) = \sum_{i<j} \underline{\bar{3}}(A_{ij})|1/2(S_{ij})\rangle \tag{5.51a}$$

$$\underline{\bar{3}}(3/2) = [\underline{\bar{3}}(A_{12}) + \underline{\bar{3}}(A_{13}) + \underline{\bar{3}}(A_{23})]|3/2\rangle \tag{5.51b}$$

For $\underline{3} \otimes \underline{1} \otimes \underline{1}$, $p \in \underline{3}$, $x, y \in \underline{1}$, we obtain, as in Section 5.4,

$$\underline{3}(1/2) \qquad \text{for} \quad pxx \tag{5.52}$$

and

$$\underline{3}(1/2, 1/2, 3/2) \qquad \text{for} \quad pxy \text{ with } x \neq y \tag{5.53}$$

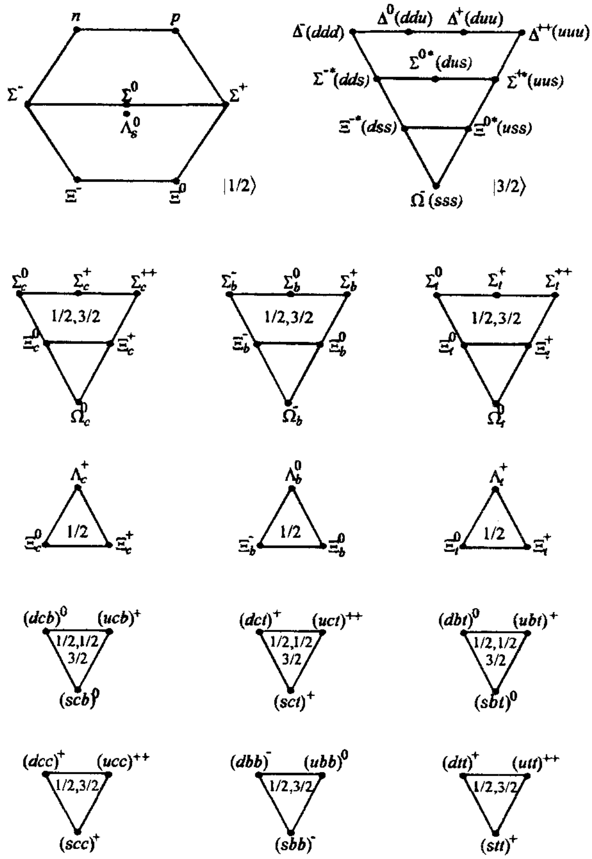
(5.34)

(5.36)

(5.37)

(5.38)
(5.39)

(5.40)



(5.41) $(cbt)_{1/2,1/2,3/2}$

(5.42) $[(ccb), (cct), (cbb), (bbt), (ctt), (btt)]_{1/2,3/2}$

(5.43) $(ccc, bbb, ttt)_{3/2}$

Fig. 7. Baryon multiplets.

where $xx = cc, bb, tt$, and $xy = cb, ct, bt$. It is not possible to form a totally a.s. flavor state to combine with $|3/2\rangle$ for pxx.

For $\underline{1} \otimes \underline{1} \otimes \underline{1}$, $x, y \in \underline{1}$, there is no way of obtaining an a.s. state with xxx, but we can construct a.s. states with

$$\underline{1}(1/2) \qquad \text{for} \quad xxy = ccb, cct, bbc, bbt, ttc, ttb \tag{5.54}$$

and

$$\underline{1}(1/2, 1/2, 3/2) \qquad \text{for } [cbt] \tag{5.55}$$

as in (5.41)–(5.44).

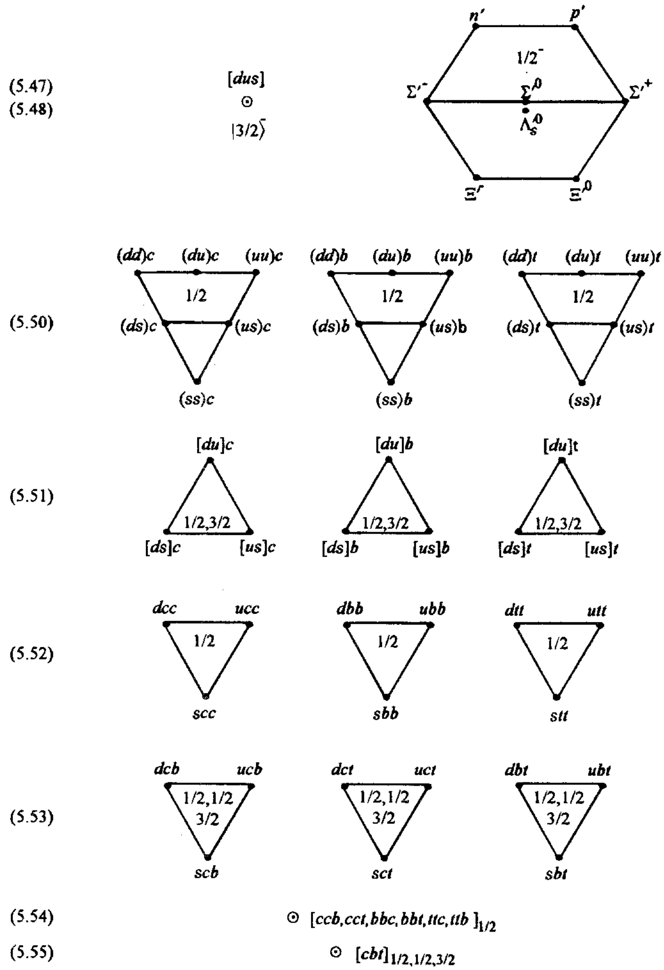


Fig. 8. Excited baryon multiplets with $\eta/\eta_0 = -1$.

Consequently, the number of spin-1/2 states is

$$8 + (3 + 6) \times 3 + (3 \times 3 \times 3) + (6 + 2) = 70$$

and the number of spin-3/2 states is

$$1 + (3 \times 3) + (3 \times 3 \times 1) + 1 = 20.$$

This is in accord with (5.45).

5.6. Hadron Excited States

Since all $6^2 = 36$ meson states are allowed in the ground state, they all have negative intrinsic parity, because they consist of $q\bar{q}$. The excited meson states then form two towers, one based on the 36 spin-0^- mesons, and the other on the 36 spin-1^- mesons. The total angular momentum (spin) increases by units of $1\hbar$ with alternating sign of parity. The correlation between angular momentum and parity is derived from the extension of the ground-state internal algebra to a dynamical algebra which yields the excited state QN (Barut, 1972, 1980a).

The $6^3 = 216$ baryon flavor states decompose into 70 spin-1/2 and 56 spin-3/2 ground states, plus 70 spin-1/2 and 20 spin-3/2 excited states of opposite parity. If we arbitrarily assign positive parity to the ground states, then we have four towers of excited states based on the states having $J^P = 1/2^+, 3/2^+; 1/2^-, 3/2^-$. The value of the angular momentum J of the states over each tower increases by units of $1\hbar$, with alternating sign of parity.

6. CONCLUSIONS

Three models of the first type, in which particles are considered to be dynamically bound systems of a few basic physical constituents, were presented: the stable particles model in Section 2, the lN-model in Section 3, and the $l\Lambda'$-model in Section 4. They all allow too many baryon states that must be eliminated by a detailed study of the interactions of the constituents.

The present quark model of hadrons, which considers hadrons to be dynamically bound systems of current quarks plus an infinite sea of quark–antiquark pairs, gluons, and Higgs particles, also has many serious difficulties (Raczka, 1993).

It is thus worthwhile to consider a model of the second type, in which particle states are constructed from tensor products of "symmetry constituents" that are basis elements of finite IRs of an internal algebra $\mathcal{A}$, and need not represent physical particles. They are completely specified by the QNs of $\mathcal{A}$ and spin, and are not assigned any other physical properties such as mass, magnetic moment, momentum, or position. All the physical properties belong to the hadron compound system.

The lq-model of Section 5 is a model of this type. All observed hadron ground states are in accord with its predictions, and it can serve as a valuable guide for the prediction and classification of new hadrons. Its main features are:

1. The internal algebra is $\mathcal{A} = su(4)_c \times su(6)_f$.
2. The color algebra $su(4)_c$ unites the quark color-triplet with the lepton color-singlet in one FIR $\underline{4}_c$.

3. The lepton states are specified by the QNs of three disjoint $su(2)$ subalgebras of $su(6)_f$ that correspond to the three lepton generations (e^-, ν_e), (μ^-, ν_μ), and (τ^-, ν_τ).

4. The quark states are specified by the QNs of $su(6)_f$ and its subalgebras $su(5)_f$, $su(4)_f$, $su(3)_f$, and $su(2)_f$. With respect to $su(3)_f$, they form the original triplet (d, u; s), and three singlets c, b, and t. They do *not* form the three doublets (d, u), (s, c), and (b, t) of current quark theory.

5. A complete analysis of the $su(5)_f$ and $su(6)_f$ hadron multiplets is given, which, to our knowledge, is new.

6. The numbers of hadron ground states are 36 spin-0 mesons, 36 spin-1 mesons, 70 spin-1/2 baryons, and 56 spin-3/2 baryons.

7. The lowest-lying excited baryon states having opposite parity to the ground-state baryons are 70 with spin 1/2 and 20 with spin 3/2.

8. The vector boson states are constructed from the tensor products of lepton and antilepton states as a charge isospin-singlet, which we identify with the photon γ, and an isospin-triplet, which we identify with $W^{\pm}$ and Z^0 [see (5.23)].

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